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# Supersymmetry of tensionless rotating strings in $AdS_5 \times S^5$, and nearly-BPS operators

**David Mateos**
*Perimeter Institute for Theoretical Physics*
*Waterloo, Ontario N2J 2W9, Canada*
*E-mail:* dmateos@perimeterinstitute.ca

**Toni Mateos**
*Departament ECM, Facultat de Física*
*Universitat de Barcelona, Institut de Física d'Altes Energies, and*
*CER for Astrophysics, Particle Physics and Cosmology*
*Diagonal 647, E-08028 Barcelona, Spain*
*E-mail:* tonim@ecm.ub.es

**Paul K. Townsend**
*Department of Applied Mathematics and Theoretical Physics*
*Centre for Mathematical Sciences*
*Wilberforce Road, Cambridge CB3 0WA, U.K.*
*E-mail:* p.k.townsend@damtp.cam.ac.uk

Abstract: It is shown that a class of rotating strings in $AdS_5 \times S^5$ with SO(6) angular momenta $(J, J', J')$ preserve 1/8-supersymmetry for large $J, J'$, in which limit they are effectively tensionless; when $J = 0$, supersymmetry is enhanced to 1/4. These results imply that recent checks of the AdS/CFT correspondence actually test a nearly-BPS sector.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence, Supersymmetry and Duality.




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## Contents



## 1. Introduction

The AdS/CFT correspondence states that $\mathcal{N} = 4$ super Yang-Mills (SYM) theory is the holographic dual of IIB string theory on $AdS_5 \times S^5$ [1, 2, 3]. The latter is usually said to reduce to supergravity in a weak-coupling and infinite-tension limit. However, this limit also validates a semi-classical quantization of macroscopic strings [4]. Examples of these that will be of interest here are provided by closed strings on $S^5$ that lie at the origin of $AdS_5$ and are supported against collapse by their SO(6) angular momenta [5, 6, 7].

Closed strings in flat space that are supported by angular momentum were first found by Hoppe and Nicolai [8], and their stability established in [9]. These Hoppe-Nicolai configurations actually lie on a sphere, so it is not surprising that they also solve the equations of motion when flat space is replaced by $AdS_5 \times S^5$, as was discovered by Frolov and Tseytlin (FT) [5], who also found a more general type of configuration supported by two independent angular momenta. An analysis of the stability of these rotating string solutions has revealed that they are typically unstable.[1] Despite this, spectacular agreement has been found between their energies and the anomalous dimensions of the corresponding CFT dual operators, in the limit of large angular momenta [10, 7, 11]. It has been suggested that this is a test of the AdS/CFT correspondence in a 'far-from-BPS sector'.

It is true that the FT rotating strings break all supersymmetries, but an inspection of the stability results of these authors shows that any instability disappears in the limit of infinite angular momentum. Furthermore (as will be shown) a BPS-type bound on the energy is saturated in this limit. This suggests that the FT rotating string solutions might

[1] As the quadratic fluctuations depend on the curvature of the ambient space, the stability properties in $AdS_5 \times S^5$ can be quite different from those in flat space.

become supersymmetric in this limit, in which case the test of the AdS/CFT correspondence that they provide would actually be in a 'nearly-BPS' sector. We say 'nearly-BPS' instead of 'near-BPS' because the limit of infinite angular momentum is also a limit of infinite energy and cannot actually be taken; there is no supersymmetric rotating string solution 'near to' the solutions of Frolov and Tseytlin. Nevertheless, the infinite-energy limit is an ultra-relativistic limit in which the string is effectively tensionless and, as we will show, this tensionless string *is* supersymmetric. Since an FT string at large angular momentum is 'nearly tensionless' it is also 'nearly supersymmetric'.

We establish this result using the by now standard procedure involving the '$\kappa$-symmetry' transformations of the IIB Green-Schwarz superstring fermions. This computation reveals that a tensionless FT string preserves 1/4 or 1/8 of the supersymmetry, according to whether the rank of the angular-momentum two-form is four or six, respectively. Because $AdS_5 \times S^5$ is conformally flat, it might appear that there is no difference between $AdS_5 \times S^5$ and flat space for a tensionless string. However, $AdS_5 \times S^5$ is not *superconformally* flat [12], and indeed the supersymmetric configurations that we find in $AdS_5 \times S^5$ are *not* supersymmetric in flat space.

## 2. Rotating strings

We will be considering solutions of the classical equations of motion derived from the Nambu-Goto Lagrangean

$$\mathcal{L} = -\frac{1}{\alpha'}\sqrt{-\det g}\,, \tag{2.1}$$

where $g$ is the worldsheet metric induced from the $AdS_5 \times S^5$ spacetime metric. We begin with a review of the relevant rotating string solutions of [5], which are strings that lie at the origin of $AdS_5$ and rotate only in the $S^5$, so effectively they live in a $\mathbb{R} \times S^5$ subspace of $AdS_5 \times S^5$ with metric

$$ds^2 = R^2\left(-dt^2 + d\Omega_5^2\right), \tag{2.2}$$

where the constant $R$ is related to the inverse string tension, $\alpha'$, and the 't Hooft coupling, $\lambda$, by $R^2 = \alpha'\sqrt{\lambda}$, and $d\Omega_5^2$ is the SO(6)-invariant metric on the unit five-sphere. The latter may be viewed as the submanifold $|\mathbf{W}| = 1$ of $\mathbb{R}^6$ with cartesian coordinates $W_i$ ($i = 1, \ldots, 6$). For the parametrization with

$$\begin{aligned} W_1 + iW_2 &= \cos\theta\, e^{i\chi}\,, \\ W_3 + iW_4 &= \sin\theta\,\cos\phi\, e^{i\alpha}\,, \\ W_5 + iW_6 &= \sin\theta\,\sin\phi\, e^{i\beta}\,, \end{aligned} \tag{2.3}$$

this gives

$$d\Omega_5^2 = d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\, d\chi^2 + \sin^2\theta\cos^2\phi\, d\alpha^2 + \sin^2\theta\sin^2\phi\, d\beta^2\,. \tag{2.4}$$

We fix the worldvolume reparametrization invariance by the gauge choice

$$t = \tau\,, \qquad \alpha = \sigma\,, \tag{2.5}$$

where $\tau$ and $\sigma$ are the worldsheet coordinates.[2] The string solutions of interest correspond to circular, rotating strings supported against collapse by their angular momenta; they are given by

$$\theta = \theta_0 \,, \qquad \chi = \nu\tau \,, \qquad \phi = \omega\tau \,, \qquad \beta = \sigma \,, \tag{2.6}$$

where $\theta_0$ is a constant in the interval $[0, \pi/2]$. At any instant the string is a circle in a two-plane contained within the 3456-space; this plane rotates with angular velocity $\omega$. In turn, the string's centre of mass rotates with angular velocity $\nu$ around a circle in the 12-plane.

Under the above circumstances, the Nambu-Goto equations are solved if either of the following relations hold[3]

$$\begin{aligned} &\text{(i)} \quad \cos\theta_0 = 0 \,, \qquad \omega^2 < 1 \,, \\ &\text{(ii)} \quad \cos 2\theta_0 = \frac{\omega^2 - 1}{\omega^2 - \nu^2} \,, \qquad \nu^2 < 1 \,, \qquad 2\omega^2 - \nu^2 - 1 > 0 \,. \end{aligned} \tag{2.7}$$

The restrictions on the angular velocities follow from demanding the reality of both $\mathcal{L}$ and $\theta_0$. The energy of the rotating string is

$$E = \sqrt{\lambda}\, |\sin\theta_0|\, \Delta^{-1/2} \,, \qquad \Delta \equiv 1 - \nu^2 \cos^2\theta_0 - \omega^2 \sin^2\theta_0 \,, \tag{2.8}$$

while the only non-zero components of the angular momentum two-form, $J_{ij}$, in the cartesian coordinates $W_i$, are

$$J \equiv J_{12} = E\, \nu \cos^2\theta_0 \,, \qquad J' \equiv J_{35} = J_{46} = \frac{1}{2} E\, \omega \sin^2\theta_0 \,. \tag{2.9}$$

Thus, $J$ and $2J'$ are the momenta conjugate to $\chi$ and $\phi$, respectively. Their values for the two possible solutions of the equations of motion are

$$\begin{aligned} &\text{(i)} \quad E = \frac{\sqrt{\lambda}}{\sqrt{1-\omega^2}} \,, \qquad J = 0 \,, \qquad J' = \frac{\sqrt{\lambda}\,\omega}{\sqrt{1-\omega^2}} \,, \\ &\text{(ii)} \quad E = \frac{\sqrt{\lambda}}{\sqrt{\omega^2 - \nu^2}} \,, \qquad J = \sqrt{\lambda}\, \frac{\left(2\omega^2 - \nu^2 - 1\right)\, \nu}{2\left(\omega^2 - \nu^2\right)^{3/2}} \,, \qquad J' = \sqrt{\lambda}\, \frac{\omega\left(1 - \nu^2\right)}{2\left(\omega^2 - \nu^2\right)^{3/2}} \,. \end{aligned} \tag{2.10}$$

In the first case it is easy to express the energy solely as a function of the angular momentum, with the result

$$(i) \quad E = \sqrt{(2J')^2 + \lambda} = |2J'| \left[ 1 + \mathcal{O}\left( \frac{\lambda}{J'^2} \right) \right] . \tag{2.11}$$

[2]Note that in our conventions both the spacetime and the worldsheet coordinates are dimensionless, which implies that the energy of the string, $E$, is also dimensionless. The corresponding dimensionful time and energy are obtained by rescaling the dimensionless ones by appropriate powers of $R$.

[3]Note that in case $(i)$ the string always lies at the origin of the 12-plane, which is a singular submanifold of the coordinate system we are using; $\nu$ is the angular velocity in a circle of zero radius. In practice this does not cause a problem because $\nu$ is irrelevant in this case.



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The expression for $E$ in terms of $J$ and $J'$ can also be found explicitly in the second case, but the result is rather messy. However, when expanded for large $J, J'$, it yields

$$(ii) \quad E = \left(|J| + |2J'|\right) \left[1 + \mathcal{O}\left(\frac{\lambda}{J^2}, \frac{\lambda}{J'^2}\right)\right]. \tag{2.12}$$

In either case, the leading, $\lambda$-independent terms saturate the BPS bounds on the energy that we will derive from the $AdS_5 \times S^5$ superalgebra in the following section. Thus the bounds are asymptotically saturated in the limit of large angular momenta. This result suggests that the circular, rotating strings become supersymmetric in this limit; we will confirm this by an explicit calculation in section 4.

At first sight, our results seem to conflict with the analysis of perturbative stability of these solutions in [5, 6]. It was shown there that the solutions $(i)$ are unstable if $J > 3\sqrt{\lambda}/8$, and that those of $(ii)$ are only stable if $J > 2J'/3$. These instabilities are related to some fluctuations about the solutions having tachyonic masses.[4] However, these masses scale as $m^2 \sim -E^{-2}$, where $E$ is the energy of the classical solution. In the large angular momenta limit, which is also a large energy limit, these masses tend to zero, so that all instabilities disappear asymptotically.

## 3. BPS Bound from the superalgebra

The energy of supersymmetric string states in $AdS_5 \times S^5$ must saturate a BPS bound that follows from the PSU$(2,2|4)$ isometry superalgebra of the $AdS_5 \times S^5$ vacuum, and hence their energy can be expressed as a function of their charges alone. In this section we review the BPS bound for states that carry the same type of charges as the rotating, circular strings above, that is, energy and angular momenta on the $S^5$. The BPS bound we will derive may be equally well understood as a statement about the supersymmetry properties of operators in the dual CFT; we will come back to this point in the penultimate section.

Let $\gamma_m$ ($m = 0, \ldots, 4$) be the $4 \times 4$ five-dimensional Dirac matrices for $AdS_5$ and let $Q^i$ be the four $AdS_5$ Dirac spinor charges, transforming as the **4** of SU(4). The non-zero anticommutators are

$$\{Q^i, Q^\dagger_j\} = \gamma^0 \left[\left(\gamma_m P^m + \frac{1}{2}\gamma_{mn} M^{mn}\right) \delta^i{}_j + 2\mathbb{I}\, B^i{}_j\right], \tag{3.1}$$

where $P, M$ are the $AdS_5$ charges and $B$ is the hermitian traceless matrix of SU(4) charges. For our spinning string configurations the only non-zero AdS charge is the energy $P^0 = E$; in this case (3.1) reduces to

$$\{Q^i, Q^\dagger_j\} = \mathbb{I}\, \delta^i{}_j E + 2\gamma^0 B^i{}_j\,. \tag{3.2}$$

By means of an SU(4) transformation we may bring $B$ to diagonal form with diagonal entries $b_i$ ($i = 1, 2, 3, 4$) satisfying

$$b_1 + b_2 + b_3 + b_4 = 0\,. \tag{3.3}$$

[4] The relevant tachyonic masses appear in formulas (4.34) of [5] and (2.35), (2.36) of [6].

The eigenvalues of the $16 \times 16$ matrix $\{Q, Q^\dagger\}$ are therefore $E \pm 2b_1, E \pm b_2, E \pm b_3, E \pm b_4$, each being doubly degenerate. Since this matrix is manifestly positive in any unitary representation, unitary implies the bound

$$E \geq 2b\,, \quad b = \sup\{|b_1|, |b_2|, |b_3|, |b_4|\}\,. \tag{3.4}$$

When the bound is saturated the matrix $\{Q, Q^\dagger\}$ will have zero eigenvalues; the possible multiplicities are $2, 4, 8, 16$. The maximum number (16) occurs when $b_i = b$ for all $i$, in which case (3.3) implies $b = 0$ and hence $E = 0$; this is the $adS_5$ vacuum. Otherwise, one has preservation of $1/8, 1/4, 1/2$ supersymmetry when $\{Q, Q^\dagger\}$ has $2, 4, 8$ zero eigenvalues, respectively.

The eigenvalues of $B$ are SU(4) invariants and hence determined in any SU(4) irrep by that irrep's Dynkin labels $(d_1, d_2, d_3)$. Conversely, the Dynkin labels are determined by the eigenvalues of $B$, and consideration of the highest weight state leads to the relation

$$d_1 = b_1 - b_2\,, \qquad d_2 = b_2 - b_3\,, \qquad d_3 = b_3 - b_4\,. \tag{3.5}$$

Given the constraint (3.3), this can be inverted to give

$$\begin{aligned} b_1 &= \frac{1}{4}(3d_1 + 2d_2 + d_3)\,, \\ b_2 &= \frac{1}{4}(-d_1 + 2d_2 + d_3)\,, \\ b_3 &= \frac{1}{4}(-d_1 - 2d_2 + d_3)\,, \\ b_4 &= \frac{1}{4}(-d_1 - 2d_2 - 3d_3)\,. \end{aligned} \tag{3.6}$$

The SU(4) charges of the spinning strings considered here correspond to irreps with Dynkin labels [5]

$$[d_1, d_2, d_3] = [J' - J, 0, J + J'] \qquad \text{if } J' > J\,, \tag{3.7}$$

$$[d_1, d_2, d_3] = [0, J - J', 2J'] \qquad \text{if } J \leq J'\,. \tag{3.8}$$

It follows, for either case, that the four (unordered) eigenvalues of $B$ are

$$\left\{J' - \frac{1}{2}J\,, \frac{1}{2}J\,, \frac{1}{2}J\,, -J' - \frac{1}{2}J\right\}. \tag{3.9}$$

Using this in (3.4), we deduce that

$$E \geq |J| + 2|J'|\,. \tag{3.10}$$

When this bound is saturated the matrix of anticommutators of supersymmetry charges will have zero eigenvalues, corresponding to the preservation of some fraction of supersymmetry. Let us determine this fraction under the assumption that $J' > J$. In this case the Dynkin labels are given by (3.7) and hence

$$b_1 = J' - \frac{1}{2}J\,, \qquad b_2 = \frac{1}{2}J\,, \qquad b_3 = \frac{1}{2}J\,, \qquad b_4 = -J' - \frac{1}{2}J\,. \tag{3.11}$$

Generically, $|b_4| = b$ and all other eigenvalues of $B$ have absolute value less than $b$ so the supersymmetry fraction is 1/8. However, this fraction is enhanced to 1/4 if $J = 0$ because then $|b_1| = |b_4| = b$ with $|b_2|, |b_3| < b$.

A similar analysis for $J \geq J'$ again yields the fraction 1/8 generically, with enhancement to 1/2 if $J' = 0$; in this case the string reduces to a point-like string orbiting the $S^5$ along an equator, as considered by Berenstein, Maldacena and Nastase (BMN) [13]. Finally, if $J = J' = 0$ then $b_i = 0$ for all $i$, $E = 0$, and all supersymmetries are preserved, as expected for the $AdS_5 \times S^5$ vacuum.

## 4. Supersymmetry

The supersymmetries preserved by a IIB string correspond to complex Killing spinors $\epsilon$ of the background that satisfy[5]

$$\Upsilon\, \epsilon = \sqrt{-\det g}\, \epsilon\,, \qquad \Upsilon = X'^M \dot{X}^N \gamma_{MN}\, K\,, \tag{4.1}$$

where $K$ is the operator of complex conjugation, and $\gamma_M$ are the (spacetime-dependent) Dirac matrices.

Recall that we are interested in strings that live in the $\mathbb{R} \times S^5$ submanifold of $AdS_5 \times S^5$ with metric (2.4), and whose embedding is specified by equation (2.6). Under these circumstances[6]

$$\sqrt{-\det g} = \sin\theta \sqrt{1 - a^2 - b^2}\,, \qquad \dot{X} \cdot \gamma = \Gamma_t + a\,\Gamma_\chi + b\,\Gamma_\phi\,, \tag{4.2}$$

where $\Gamma_\theta, \Gamma_\phi, \ldots$ are ten-dimensional tangent space (i.e. constant) Dirac matrices, and

$$a = \nu\, \cos\theta\,, \qquad b = \omega\, \sin\theta\,. \tag{4.3}$$

Note that the lorentzian signature of the induced worldsheet metric implies that

$$a^2 + b^2 \leq 1\,. \tag{4.4}$$

The Killing spinors of $AdS_5 \times S^5$ restricted to the relevant submanifold take the form

$$\epsilon = e^{\frac{t}{2} i \tilde{\Gamma}}\, e^{\frac{\theta}{2} i \gamma_* \Gamma_\theta}\, e^{\frac{\phi}{2}\Gamma_{\theta\phi}}\, e^{\frac{\chi}{2} i \gamma_* \Gamma_\chi}\, e^{\frac{\alpha}{2}\Gamma_{\theta\alpha}}\, e^{\frac{\beta}{2}\Gamma_{\phi\beta}}\, \epsilon_0\,, \tag{4.5}$$

where $\epsilon_0$ is a constant spinor, $\gamma_* = \Gamma_{\theta\phi\chi\alpha\beta}$, and $\tilde{\Gamma}$ is a constant matrix that commutes with all other matrices above (its specific form will not be needed). In our conventions all these matrices are real. For our configurations $\dot{X} \cdot X' = 0$, so the supersymmetry preservation condition (4.1) can be written as

$$\left(X' \cdot \gamma\right) \left(\dot{X} \cdot \gamma\right) \epsilon = -\sqrt{-\det g}\; K\, \epsilon\,. \tag{4.6}$$

[5] Thus, $\Upsilon / \sqrt{-\det g}$ is the matrix $\Gamma_\kappa$ appearing in the kappa-symmetry transformation of the fermionic variables of the Green-Schwarz IIB superstring.

[6] As the radius $R$ cancels in the final result we set $R = 1$ in this section.



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This must be satisfied for all $\tau, \sigma$, but it is useful to first consider $\tau = 0$, in which case it reduces to

$$\sin\theta \left(\dot{X} \cdot \gamma\right) \Gamma_\alpha K e^{\frac{\theta}{2} i\gamma_* \Gamma_\theta} e^{\frac{\sigma}{2}(\Gamma_{\theta\alpha} + \Gamma_{\phi\beta})} \epsilon_0 = \sqrt{-\det g}\, e^{\frac{\theta}{2} i\gamma_* \Gamma_\theta} e^{\frac{\sigma}{2}(\Gamma_{\theta\alpha} + \Gamma_{\phi\beta})} \epsilon_0 \,. \tag{4.7}$$

It can be shown that in order for this equation to be satisfied for all $\sigma$, one must impose

$$\Gamma_{\theta\alpha\phi\beta}\, \epsilon_0 = \epsilon_0 \,, \tag{4.8}$$

in which case the equation becomes

$$\left[\Gamma_t + (\cos\theta - \sin\theta\, i\gamma_* \Gamma_\theta)\,(a\,\Gamma_\chi + b\,\Gamma_\phi)\right] \Gamma_\alpha K \epsilon_0 = \sqrt{1 - a^2 - b^2}\, \epsilon_0 \,. \tag{4.9}$$

Equations (4.8) and (4.9) are equivalent to the two equations

$$A\,\epsilon_0 = \epsilon_0 \,, \qquad A \equiv a\, \cos\theta\, \Gamma_{t\chi} + b\, \sin\theta\, i\Gamma_{t\chi\alpha\beta} \,, \tag{4.10}$$

$$B\,K\epsilon_0 = \sqrt{1 - a^2 - b^2}\, \epsilon_0 \,, \qquad B \equiv a\, \sin\theta\, i\Gamma_{\phi\beta} + b\, \cos\theta\, \Gamma_{\phi\alpha} \,. \tag{4.11}$$

Given that $a$ and $b$ are non-zero, it follows from (4.10) that

$$i\Gamma_{\alpha\beta}\, \epsilon_0 = s\, \epsilon_0 \,, \qquad \Gamma_{t\chi}\, \epsilon_0 = s\, \epsilon_0 \,, \tag{4.12}$$

and

$$a\, \cos\theta + s\, b\, \sin\theta = \tilde{s} \,, \tag{4.13}$$

where $s$ and $\tilde{s}$ are independent signs. The latter relation is compatible with the restriction (4.4) if and only if $b\, \cos\theta = s\, a\, \sin\theta$, and these two relations for $a$ and $b$ imply, given (4.3), that

$$\nu = \tilde{s} \,, \qquad \omega = s\, \tilde{s} \,. \tag{4.14}$$

It then follows that the equation (4.11) is trivially satisfied, and that $\sqrt{-\det g} = 0$. The string worldsheet must therefore be null, which is only possible for a tensionless string. Although the IIB superstring is not tensionless, the energy due to the rotation is much greater than the energy due to the tension in the limit of large angular momentum. So in this limit the string is effectively tensionless.

We continue now by considering only the tensionless string, for which the supersymmetry preserving condition (4.6) reduces to

$$\left(\dot{X} \cdot \gamma\right) \epsilon = 0 \,. \tag{4.15}$$

The analysis of this equation for $\tau = 0$ reproduces the results already obtained from an analysis of (4.7), which are summarized by the projections

$$\Gamma_{\theta\alpha\phi\beta}\, \epsilon_0 = \epsilon_0 \,, \qquad \Gamma_{t\chi}\, \epsilon_0 = \omega\nu\, \epsilon_0 \,, \qquad i\Gamma_{\alpha\beta}\, \epsilon_0 = \omega\nu\, \epsilon_0 \,. \tag{4.16}$$

It is now straightforward to check that a spinor $\epsilon_0$ satisfying these conditions solves (4.15) for all $\tau$ and $\sigma$. It thus follows that the generic null FT string preserves 1/8 of the 32 supersymmetries of the IIB $AdS_5 \times S^5$ vacuum.

We have assumed above that $a$ and $b$ are non-zero. A solution with $b = 0$ has $J' = 0$ and corresponds to a point-like, collapsed string moving along a great circle of $S^5$, as considered in [13], whereas a solution with $a = 0$ has $J = 0$. Redoing the analysis it is easy to see the former preserve 1/2 of the supersymmetry. Similarly, in the second case one finds that the necessary and sufficient conditions for preservation of supersymmetry are

$$\omega^2 = 1\,, \qquad \cos\theta_0 = 0\,, \tag{4.17}$$

and that the projections on $\epsilon$ are

$$\Gamma_{\theta\alpha\phi\beta}\,\epsilon_0 = \epsilon_0\,, \qquad i\Gamma_{t\chi\alpha\beta}\epsilon_0 = \omega\epsilon_0\,. \tag{4.18}$$

These projections preserve 1/4 of the thirty-two supersymmetries of the IIB $AdS_5 \times S^5$ vacuum.

## 5. Nearly-BPS operators

Macroscopic, rotating strings in $AdS_5 \times S^5$ with SO(6) angular momenta are expected to be dual, at least in the limit of large angular momenta, to operators of $\mathcal{N} = 4$ SYM theory in appropriate SO(6) representations, with the energy of each string equal to an eigenvalue of the matrix of anomalous dimensions. The rotating strings considered here are expected to correspond to operators transforming in SO(6) representations with Dynkin labels as in (3.7), (3.8). The highest-weight operators in these representations are linear combinations of traces, or products of traces, of the scalar operators

$$X \equiv W_1 + iW_2\,, \qquad Y \equiv W_3 + iW_5\,, \qquad Z = W_4 + iW_6\,, \tag{5.1}$$

associated to the non-zero components of the string's angular momenta, $J_{12}, J_{35}$ and $J_{46}$.

For $J, J' \ll \sqrt{N}$, the $p$-trace operators are (approximately) orthogonal to the $q$-trace operators for $p \neq q$. In this limit[7] an $n$-string state is associated with an $n$-trace operator. As we consider a single string, we expect the SO(6) highest-weight dual operator to be a single-trace operator of the form [5]

$$\mathcal{O}(J, J') = \mathrm{Tr}\left(X^J Y^{J'} Z^{J'}\right) + \cdots\,, \tag{5.2}$$

where the dots stand for the permutations of the factors needed so that the operator transforms in the appropriate irrep of SO(6). Evidence for this correspondence is that the anomalous dimensions of the $\mathcal{O}$-type operators have been computed by spin-chain methods in the one-loop planar approximation [10], and perfect agreement has been found with the string prediction in the large angular momenta limit [5, 6, 7]. Note that the spin chain computation implicitly assumes that $J, J' \ll \sqrt{N}$, because this condition is needed to justify the restriction to planar diagrams; as in the BMN case, non-planar diagrams are expected to be suppressed by powers of $J^2/N$, $J'^2/N$.

[7] The assumption that $J, J' \ll \sqrt{N}$ is compatible with our other assumption that $J, J' \gg \sqrt{\lambda}$, and the two together imply that the IIB string theory is weakly coupled.



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Our results concerning the supersymmetry of the rotating strings dual to the $\mathcal{O}$-type operators in the limit of large angular momenta imply that these operators are 'nearly-BPS' in this limit, in a sense that we now aim to clarify. Given that anomalous dimensions of these operators are known only for $J, J' \ll \sqrt{N}$, we shall continue to assume, for the moment, that the same condition holds on the string theory side of the correspondence, and we begin by summarising what it means for an operator to be a BPS operator. All primary operators in a superconformal multiplet can be obtained by the action of the $Q$-supersymmetry charges on a lowest-dimension operator, a so-called *super*conformal primary operator, which commutes with all the $S$-supersymmetry charges. The superconformal algebra implies that a superconformal primary operator also commutes with some $Q$-supersymmetry charges if and only if its conformal dimension saturates an appropriate BPS bound in terms of its rotational and R-symmetry quantum numbers (this is equivalent to the bound on the adS energy that we derived in section 3). If this happens the corresponding supermultiplet is shorter than a generic supermultiplet, and each operator in a shortened multiplet is called a BPS operator. Note that merely commuting with some Poincaré supersymmetry charges does not make an operator a BPS operator.

Now we examine the $\mathcal{O}$-type operators of (3.4). These are primary (after diagonalisation of the matrix of anomalous dimensions) but not superconformal primary; for example, the operator with $J = 0$, $J' = 2$ is a descendant of the Konishi operator. Moreover, they are not 1/4-BPS or 1/8-BPS operators, since in $\mathcal{N} = 4$ SYM such BPS operators are linear combinations of multi-trace operators that involve at least[8] a double-trace or a triple-trace operator, respectively [14, 15, 16]. Therefore, although $\mathcal{O}$ is a nearly-BPS operator in the sense that its conformal dimension almost saturates a BPS bound when $\lambda/J^2, \lambda/J'^2 \ll 1$, it is not the case that $\mathcal{O}$ approximates an exact 1/4-BPS or 1/8-BPS operator in this regime. In this sence the $\mathcal{O}$-type operators are not 'near-BPS', but they are effectively so for any computation that depends only on the conformal dimension and R-symmetry quantum numbers. We call these operators 'nearly-BPS'.

To actually take the limit $\lambda/J^2, \lambda/J'^2 \to 0$ we would need to go to the free theory, $\lambda = 0$. In this case, the conformal dimension of $\mathcal{O}$ does saturate a BPS bound, and therefore must belong to a shortened supermultiplet. This is possible because operators that are descendents of superconformal primaries in the interacting theory can become independent BPS operators in the free-field limit [17, 18]. Note that there will be as many of these additional shortened multiplets as are required to form a long one, so the shortening provides no protection against the generation of large anomalous dimensions: the usual claim that BPS-operators have protected conformal dimensions is not true without qualification.

What happens when the condition $J, J' \ll \sqrt{N}$ is not satisfied? On the field theory side, one needs to go beyond the planar approximation. Moreover, single-trace operators are no longer orthogonal to multi-trace operators. On the string theory side, provided $g_s \ll 1$, single-string states remain orthogonal to multi-string states. However, the description in terms of elementary strings is likely to be inadequate. This is indeed the case for

[8]In the free theory, $\lambda = 0$, there exist purely double-trace 1/4-BPS and purely triple-trace 1/8-BPS operators [14].

states with $J' = 0$, for which the correct semiclassical description is known to be in terms of non-perturbative, rotating, spherical D3-branes, the so-called 'giant gravitons' [19]. The operators dual to these states are not single-trace operators, but (sub)determinant operators [20]; the latter *are* approximately orthogonal to each other if $J$ is comparable to $N$, and only those with $J \leq N$ are independent from each other. A similar situation will presumably hold when both $J$ and $J'$ are non-zero. If this is the case, then the fact that the $\mathcal{O}$-type operators are only independent if $2J' + J \leq N$ (since otherwise they can be expressed as sums of products of operators of the same type) will be irrelevant to the comparison with string theory, since these operators will only provide an accurate dual description of the corresponding string theory states if $J, J' \ll \sqrt{N}$.

## 6. Discussion

Quantitative tests of the AdS/CFT conjecture that go beyond kinematics are rare because a weak-coupling computation on one side generally corresponds to an strong-coupling computation on the other side. An exception to this state of affairs occurs in the sector of the rotating strings considered here, for two reasons [5, 6, 7, 21]. First, the energy of the corresponding *classical* string configurations happens to admit an expansion in *positive* powers of $\lambda/(J + 2J')^2$. Second, partial cancellations of sigma-model quantum corrections imply that all such corrections containing non-positive powers of $\lambda$ are suppressed in the limit $J + 2J' \gg 1$. We would like to stress that, although these two facts justify a *comparison* of the string calculation to a perturbative SYM calculation, they do not imply *agreement* between the two; the latter is presumably a success of AdS/CFT.

If $J' = 0$ the strings considered here reduce to the BMN strings, that is, to point-like strings orbiting the $S^5$ around an equator, with angular momentum $J$ [13]. The dual BMN operators are near-BPS operators, in the sense that they are 'close to' (that is, 'a few impurities away from') an exactly 1/2-supersymmetric operator, the so-called BMN ground-state; thus, in the BMN case, the agreement tests the AdS/CFT conjecture in a near-supersymmetric sector, and this fact is presumably responsible for the partial cancellations of sigma-model quantum corrections that are essential for the comparison to be possible.

It has not been appreciated previously that the situation is very similar for the rotating strings discussed here with $J' \neq 0$. This is implied by the results of this paper, since we have shown that these strings asymptotically become 1/4- or 1/8-supersymmetric in the limit $\lambda/(J + 2J')^2 \ll 1$. One difference between the extended strings and the BMN collapsed strings is, however, that the operators dual to the strings with $J' \neq 0$ are not near-BPS[9] but *nearly*-BPS, in the sense that there is no exactly 1/4- or 1/8-BPS operator that these operators are close to. Moreover, the ratio between the anomalous dimension of a BMN operator and its classical dimension is $(\Delta - J)/J = \mathcal{O}(1/J)$ for large $J$ [13], whereas the corresponding ratio for an operator dual to an FT string is $(\Delta - (J + 2J'))/(J + 2J') = \mathcal{O}(\lambda/(J+2J')^2)$. Hence, for large angular momentum, the BMN operators almost saturate

[9]Except if $J' \ll J$, in which case they are BMN operators.

the BPS bound irrespectively of the value of the effective 't Hooft coupling $\lambda/J^2$. Instead, the operators dual to the FT strings almost saturate the bound, and are therefore nearly-BPS, only if this coupling is small, that is, in the perturbative regime of the SYM theory. Any calculation of a non-perturbative contribution to the anomalous dimensions of the operators dual to the FT strings could therefore lead to a comparison beyond the nearly-BPS limit.

We would like to emphasize that tensionless strings arise in our analysis via an ultra-relativistic approximation in which the energy due to the string tension is negligible compared to the kinetic energy. In this sense, we think of the limit $\lambda/J^2 \to 0$ as a limit in which $\lambda$ is kept fixed and $J$ is sent to infinity; this is particularly natural in view of the fact that we also need $J \gg 1$. However, one can equivalently think of this limit as fixing $J$ to be much larger than unity and then sending $\lambda \to 0$. In the strict limit $\lambda = 0$ the rotating strings become exactly supersymmetric, and this may be relevant to the correspondence between tensionless strings in $AdS_5 \times S^5$ and free $\mathcal{N} = 4$ SYM theory (see, for example, [22]). Note that the free field theory has an infinite number of global symmetries, which could correspond to the gauge symmetries of massless particles of all spin in the tensionless string spectrum.[10]

In this paper we have focused on *circular* rotating strings, but the same type of agreement between string energies and conformal field theory anomalous dimensions has been found for other types of rotating strings, such as folded strings rotating in $S^5$ [7] and strings carrying angular momenta both in $S^5$ and in $AdS_5$ [23]. The natural question of whether these other types of strings also become supersymmetric in the limit of large angular momenta therefore arises. We hope to report on this in the future.

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